

Efficient atomic quantum memory for photonic qubits in cavity QED

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Abstract

We investigate a scheme of atomic quantum memory to store photonic qubits of polarization in cavity QED. It is observed that the quantum-state swapping between a single-photon pulse and a Λ -type atom can be made via scattering in an optical cavity [T. W. Chen, C. K. Law, P. T. Leung, Phys. Rev. A **69** (2004) 063810]. This swapping operates limitedly in the strong coupling regime for Λ -type atoms with equal dipole couplings. We extend this scheme in cavity QED to present a more feasible and efficient method for quantum memory combined with projective measurement. This method works without requiring such a condition on the dipole couplings. The fidelity is significantly higher than that of the swapping, and even in the moderate coupling regime it reaches almost unity by narrowing sufficiently the photon-pulse spectrum. This high performance is rather unaffected by the atomic loss, cavity leakage or detunings, while a trade-off is paid in the success probability for projective measurement.

Key words: Quantum memory; Cavity QED; Quantum information

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1 Introduction

Combined systems of atoms and photons have been studied extensively to construct promising and efficient quantum networks for information processing and communication [1]. In these quantum networks quantum-state transfer between photons and atoms (matter), and storage of quantum data are particularly important. Then, numerous methods to implement the quantum-state transfer and quantum memory have been proposed and investigated in various

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manners [2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20]. The cavity QED is among the promising schemes to realize such quantum-state operations, which utilizes strong interactions between single atoms and photons inside cavities [21,22]. Specifically, quantum-state transfer and manipulation are made between a single atom and a single-photon pulse through a scattering in an optical cavity.

In this paper we investigate a scheme of atomic quantum memory to store photonic qubits of polarization in cavity QED. It is observed that the quantum-state swapping between a single-photon pulse and a Λ -type atom can be made via scattering in an optical cavity [23]. This swapping operates limitedly in the strong coupling regime for Λ -type atoms with equal dipole couplings. We extend this scheme in cavity QED to present a more feasible and efficient method for quantum memory operation, storage and retrieval of photonic qubits, combined with projective measurement.

The present method for quantum memory has some characteristic properties and advantages as follows. First, quantum information is encoded as qubits in polarization states of single-photon pulses. Such photonic qubits are then stored in the two degenerate stable ground states (e.g., Zeeman sublevels of a hyperfine ground state) of single atoms trapped in optical cavities. Hence, this atomic quantum memory is expected to be robust against decoherence. The memory operation is performed manifestly for atomic and photonic qubits via scatterings, which is suitable for the discrete-variable quantum information scheme.

Since the quantum-state transformation for memory operation is made on the asymptotic states after scattering, no precise timing of the interaction period is required [23]. Furthermore, no control light is used for the quantum-state transfer in contrast with many other proposals for quantum memory. The present method hence operates in a passive way except for the projective measurement. This would be favorable for scalability.

The naive quantum-state swapping via atom-photon scattering operates in rather limited situations; Λ -type atoms with equal dipole couplings, and the strong coupling regime with negligible detunings [23]. In contrast, the present quantum memory operation combined with projective measurement can be implemented in more practical situations. It works without requiring such a condition on the dipole couplings of Λ -type atoms. The fidelity is significantly higher than that of the swapping, and even in the moderate coupling regime it reaches almost unity by narrowing sufficiently the photon-pulse spectrum. This high performance is rather unaffected by the atomic loss, cavity leakage or detunings, while a trade-off is paid in the success probability for projective measurement.

The rest of the paper is organized as follows. In Sec. 2, the basic description of the atom-photon scattering in cavity QED is reviewed, and the swapping between atomic and photonic qubits via scattering is discussed. In Sec. 3, by extending this scheme of quantum-state transfer in cavity QED, we present a more feasible and efficient method for quantum memory operation combined with projective measurement. A numerical analysis is presented in Sec. 4 to exhibit the high performance of the present atomic quantum memory. In Sec. 5, storage of 2-qubit photonic entanglement is also considered as an application. Section 6 is devoted to summary.

2 Atom-photon scattering and quantum-state transformation in cavity QED

We first review the basic description of the atom-photon scattering and quantum-state transformation in cavity QED [23]. A one-dimensional cavity bounded by two mirrors is used, one of which is perfectly reflecting while the other is partially transparent. The electromagnetic field is expanded in terms of the continuous modes with the wave number k , which range over the inside of cavity through the outside free space [24]. A photonic qubit is encoded in the polarization states $|k_L\rangle$ and $|k_R\rangle$ of single-photon pulse as

$$|\phi_p\rangle = c_L|\bar{k}_L\rangle + c_R|\bar{k}_R\rangle = \int_{-\infty}^{\infty} dk f(k) e^{-ikt} |\phi_p(k)\rangle, \quad (1)$$

$$|\bar{k}_{L,R}\rangle = \int_{-\infty}^{\infty} dk f(k) e^{-ikt} |k_{L,R}\rangle, \quad (2)$$

where $f(k)$ is the normalized spectral amplitude, and e^{-ikt} represents the asymptotic temporal evolution ($c = \hbar = 1$ unit). On the other hand, a Λ -type atom is trapped inside the cavity, which has two degenerate ground states $|L\rangle$ and $|R\rangle$ and an excited state $|e\rangle$. Then, an atomic qubit is encoded in the degenerate ground states as

$$|\psi_a\rangle = a_L|L\rangle + a_R|R\rangle. \quad (3)$$

The polarization states $|k_L\rangle$ and $|k_R\rangle$ are coupled, respectively, to the transitions $|L\rangle - |e\rangle$ and $|R\rangle - |e\rangle$ of a frequency ω_e in the cavity with dipole couplings

$$g_{L,R}(k) = \frac{\lambda_{L,R} \sqrt{\kappa/\pi} e^{i\theta_{L,R}}}{k - k_c + i\kappa}, \quad (4)$$

where k_c is the resonant frequency of the cavity, κ is the leakage rate of the cavity, λ_L and λ_R represent the normalized coupling strengths (single-photon Rabi frequencies), and θ_L and θ_R are the phase angles from the dipole transition matrix elements. (It will be seen later that the phase angles θ_L and θ_R are irrelevant for the transfer between atomic and photonic qubits via scattering. Actually, they may be removed at the beginning by phase transformations of $|L\rangle$ and $|R\rangle$.) The atom-photon scattering takes place through these couplings, and the transformation of the atom and photon states is made asymptotically as

$$\begin{aligned}\mathcal{T}|Lk_L\rangle &= T_{LL}(k)|Lk_L\rangle + T_{RL}(k)|Rk_R\rangle, \\ \mathcal{T}|Rk_R\rangle &= T_{LR}(k)|Lk_L\rangle + T_{RR}(k)|Rk_R\rangle, \\ \mathcal{T}|Lk_R\rangle &= |Lk_R\rangle, \\ \mathcal{T}|Rk_L\rangle &= |Rk_L\rangle,\end{aligned}\tag{5}$$

where the basis states are taken as $|Lk_L\rangle \equiv |L\rangle|k_L\rangle$ and so on. The scattering matrix elements are calculated [23] as

$$\begin{aligned}T_{LL}(k) &= e^{i\phi_s(k)} \sin^2 \xi + \cos^2 \xi, \\ T_{RR}(k) &= \sin^2 \xi + e^{i\phi_s(k)} \cos^2 \xi, \\ T_{LR}(k) &= e^{-i(\theta_L - \theta_R)} \sin \xi \cos \xi (e^{i\phi_s(k)} - 1), \\ T_{RL}(k) &= e^{i(\theta_L - \theta_R)} \sin \xi \cos \xi (e^{i\phi_s(k)} - 1),\end{aligned}\tag{6}$$

where

$$\sin \xi = \lambda_L / \lambda, \quad \cos \xi = \lambda_R / \lambda,\tag{7}$$

$$\lambda = \sqrt{\lambda_L^2 + \lambda_R^2}.\tag{8}$$

The phase shift $\phi_s(k)$ acquired by the bright state is determined independently of the photon-pulse shape $f(k)$. It is explicitly calculated as

$$e^{i\phi_s(k)} = \frac{(k - k_c + i\kappa)w_+(k - k_c)}{(k - k_c - i\kappa)w_-(k - k_c)},\tag{9}$$

where

$$w_{\pm}(s) = s^2 - (\delta_e - i\gamma \pm i\kappa)s - \lambda^2 \pm i\kappa(\delta_e - i\gamma)\tag{10}$$

with the atomic detuning $\delta_e = \omega_e - k_c$. The linear transformation \mathcal{T} via scattering with the complex $\phi_s(k)$ is generally non-unitary due to the atomic loss with a rate γ induced by the spontaneous emission into the environment.

It is observed in Ref. [23] that the quantum-state swapping between the atom and photon can be made via scattering in the strong coupling regime for a Λ -type atom having equal dipole couplings,

$$\lambda_L = \lambda_R = \lambda/\sqrt{2}. \quad (11)$$

(This condition is met, for example, by the D1 line of sodium with $|L\rangle = |F = 1, m_F = -1\rangle$, $|R\rangle = |F = 1, m_F = 1\rangle$, $|e\rangle = |F' = 1, m_{F'} = 0\rangle$.) In fact, with the maximal phase shift $e^{i\phi_s(k_c)} = -1$ at the resonance ($k = k_c = \omega_e$) in the strong coupling limit $\lambda^2/\kappa\gamma \rightarrow \infty$, we have the scattering matrix elements as $e^{i(\theta_L - \theta_R)}T_{LR}(k_c) = e^{-i(\theta_L - \theta_R)}T_{RL}(k_c) = -1$, $T_{LL}(k_c) = T_{RR}(k_c) = 0$ in Eq. (6) under the condition $\lambda_L = \lambda_R$. Then, the swapping between the atomic and photonic qubits is made as

$$|\psi_a\rangle|\phi_p(k_c)\rangle \xrightarrow{\mathcal{T}} e^{-i(\theta_L + \theta_R)}|\psi_{\text{swap}}\rangle|\phi_{\text{swap}}(k_c)\rangle, \quad (12)$$

where

$$|\psi_{\text{swap}}\rangle = c_R(e^{i\theta_R}|L\rangle) + c_L(-e^{i\theta_L}|R\rangle), \quad (13)$$

$$|\phi_{\text{swap}}(k)\rangle = a_R(-e^{i\theta_R}|k_L\rangle) + a_L(e^{i\theta_L}|k_R\rangle). \quad (14)$$

It may be expected naively that this swapping is applicable for storing photonic qubits in atomic qubits. In order to examine the feasibility of atom-photon swapping for quantum memory we evaluate the fidelity as follows. Arbitrary atomic and photonic qubits in Eqs. (1) and (3) are taken as the initial state

$$|\Phi_{\text{in}}\rangle = |\psi_a\rangle|\phi_p\rangle. \quad (15)$$

Then, the density operator of the output state via scattering is given by

$$\rho_{\text{out}} = |\Phi_{\text{out}}\rangle\langle\Phi_{\text{out}}| + (1 - \langle\Phi_{\text{out}}|\Phi_{\text{out}}\rangle)|0\rangle\langle 0| \quad (16)$$

with

$$|\Phi_{\text{out}}\rangle = \mathcal{T}|\Phi_{\text{in}}\rangle = \int_{-\infty}^{\infty} dk f(k) e^{-ikt} |\Phi_{\text{out}}(k)\rangle, \quad (17)$$

$$\begin{aligned} |\Phi_{\text{out}}(k)\rangle = & -e^{-2i\theta_R}T_{LR}(k)|\psi_{\text{swap}}\rangle|\phi_{\text{swap}}(k)\rangle \\ & + T_{LL}(k)|\psi_a\rangle|\phi_p(k)\rangle. \end{aligned} \quad (18)$$

Here, the relations $T_{LL}(k) = T_{RR}(k)$, $e^{i(\theta_L - \theta_R)}T_{LR}(k) = e^{-i(\theta_L - \theta_R)}T_{RL}(k)$ and $T_{LL}(k) - e^{i(\theta_L - \theta_R)}T_{LR}(k) = 1$ under the condition $\lambda_L = \lambda_R$ are considered. The “vacuum” term of $|0\rangle\langle 0|$ represents the loss due to the spontaneous emission, providing $\text{Tr}\rho_{\text{out}} = 1$. The output photon will eventually be absorbed by matter. Then, by taking the trace over the photon states the fidelity to obtain the desired atomic state $|\psi_{\text{swap}}\rangle$ is given by

$$F(|\phi_p\rangle) = [\langle\psi_{\text{swap}}|\text{Tr}_p[|\Phi_{\text{out}}(k)\rangle\langle\Phi_{\text{out}}(k)|]|\psi_{\text{swap}}\rangle]_f, \quad (19)$$

where $\text{Tr}_p[\rho] \equiv \langle k_L|\rho|k_L\rangle + \langle k_R|\rho|k_R\rangle$, and the average of any function $G(k)$ of k with the weight $|f(k)|^2$ is denoted as

$$[G(k)]_f \equiv \int_{-\infty}^{\infty} dk |f(k)|^2 G(k). \quad (20)$$

This fidelity is calculated by considering the relation $T_{LL}(k) - e^{i(\theta_L - \theta_R)}T_{LR}(k) = 1$ under the condition $\lambda_L = \lambda_R$ as

$$F(|\phi_p\rangle) = F_{\text{swap}} + (1 - F_{\text{swap}})|\langle\psi_{\text{swap}}|\psi_a\rangle|^2 \geq F_{\text{swap}}. \quad (21)$$

The fidelity of swapping is then given irrespective of the choice of initial state as

$$F_{\text{swap}} = \left[|\sin(\phi_s(k)/2)|^2 \right]_f (\lambda_L = \lambda_R). \quad (22)$$

Here, it should be noted that this fidelity of swapping F_{swap} is meaningful only for the case $\lambda_L = \lambda_R$ in calculating $F(|\phi_p\rangle)$ as Eq. (21) with $|T_{LR}(k)| = |\sin(\phi_s(k)/2)|$.

The atom-photon swapping may be optimized by satisfying the conditions $k_p = k_c = \omega_e$, $\kappa_p \ll \kappa$ and $\lambda^2/\kappa\gamma \gg 1$, as discussed in Ref. [23], where k_p and κ_p represent the peak position and width of the photon-pulse spectrum $f(k)$, respectively. Specifically, by including the effects of detunings $\delta_e = \omega_e - k_c$ and $\delta_p = k_p - k_c$ the fidelity of swapping F_{swap} in Eq. (22) is evaluated in the leading terms for $\kappa_p \rightarrow 0$ as

$$F_{\text{swap}} \approx 1 - 2(\kappa\gamma/\lambda^2) - [(\kappa\delta_e/\lambda^2) + (\delta_p/\kappa)]^2. \quad (23)$$

In order to obtain $F_{\text{swap}} \geq 0.99$, for instance, we need to arrange roughly $\kappa\gamma/\lambda^2 \lesssim 1/200$, $\kappa|\delta_e|/\lambda^2 \lesssim 1/10$ and $|\delta_p|/\kappa \lesssim 1/10$ unless a tuning is made as $\delta_p \approx -(\kappa/\lambda)^2\delta_e$. Hence, a rather strong atom-photon coupling is required to achieve a high fidelity. It should be mentioned further that this atom-photon swapping operates under the condition $\lambda_L = \lambda_R$ on the dipole couplings. In

general, for $\lambda_L \neq \lambda_R$ the transfer between atomic and photonic qubits is not implemented sufficiently even in the strong coupling limit $\lambda^2/\kappa\gamma \rightarrow \infty$. We can check specifically the relation

$$F(|\bar{k}_L\rangle) + F(|\bar{k}_R\rangle) = 1 + \left[|T_{LR}(k)|^2\right]_f \leq 1 + \sin^2 2\xi \quad (24)$$

by calculating the fidelity in Eq. (19) for $|\bar{k}_L\rangle$ and $|\bar{k}_R\rangle$ as the initial photonic qubit $|\phi_p\rangle$. This indicates that if $\lambda_L \neq \lambda_R$ ($\sin 2\xi < 1$) the fidelity of qubit transfer $F(|\phi_p\rangle)$ remains rather below unity for some set of qubits, which is insufficient for quantum memory to store unknown qubits.

3 Memory operation: storage and retrieval of photonic qubits with projective measurements

We here present an extended scheme for quantum memory via scattering in cavity QED, which works more efficiently with high fidelity in practical situations. We consider the sequence of storage and retrieval of photonic qubits, which may appear somewhat different from simply repeating twice the atom-photon swapping. To be general, we do not assume the condition (11) for Λ -type atoms, allowing different dipole couplings. Then, quantum memory operation can be implemented conditionally by making some projective measurements, as described in detail below. It will be seen that the fidelity of almost unity is achieved by narrowing sufficiently the spectral width of the photon pulse ($\kappa_p \ll \kappa$).

The schematic diagram for the sequence of storage and retrieval of a photonic qubit is depicted in Fig. 1. The initial state is taken specifically as

$$|\Phi_{\text{in}}\rangle = |R\rangle|\phi_{p1}\rangle|\bar{k}'_R\rangle \quad (25)$$

with

$$|\phi_{p1}\rangle = c_L|\bar{k}_L\rangle + c_R|\bar{k}_R\rangle. \quad (26)$$

(The third photon of $|\bar{k}''_L\rangle$ is omitted for simplicity.) Here, $|\phi_{p1}\rangle$ is an unknown photonic qubit to be stored and then retrieved. The atomic state is initially prepared to be $|R\rangle$, and the second photon pulse of $|\bar{k}'_R\rangle$ is injected after a time delay $\tau (\gg \kappa_p^{-1} \gg \kappa^{-1})$ to retrieve the stored qubit. We take for definiteness the same profile $f(k)$ for the photon pulses (though this need not be required precisely).

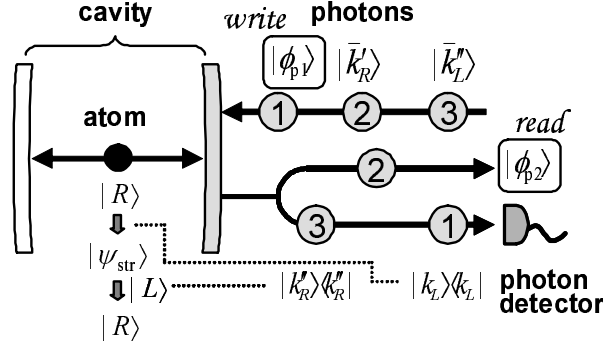


Fig. 1. The schematic diagram for the sequence of storage and retrieval of a photonic qubit via scattering combined with photon-polarization measurements. The qubit encoded in the first photon pulse is stored in the atom by detecting the output photon. Then, the second photon pulse is injected to retrieve the stored qubit. The third photon pulse is used in the retrieval process to project out the $|L\rangle$ component of the atomic state as recovering the initial $|R\rangle$ state.

Storage:

In the storage process, after the first photon pulse is scattered with the atom, the polarization “ L ” is detected on the output photon. This polarization detection is represented by a positive operator-valued measure

$$\Pi(k_L) = \int_{-\infty}^{\infty} dk \eta(k) |k_L\rangle \langle k_L| \quad (27)$$

with the quantum efficiency $0 < \eta(k) \leq 1$. [The dark count is neglected here since it can actually be made rather small. The terms of more than one photon states may also be discarded effectively in $\Pi(k_L)$ for the present process involving a single atom and a single photon.] Then, the resultant state is given by

$$\begin{aligned} \rho_1 &= \frac{1}{P(k_L)} \text{Tr}_{p1} [\Pi(k_L) \mathcal{T}_1 |\Phi_{\text{in}}\rangle \langle \Phi_{\text{in}}| \mathcal{T}_1^\dagger] \\ &= \int_{-\infty}^{\infty} dk \frac{\eta(k) |f(k)|^2}{P(k_L)} |\psi_{\text{str}}(k)\rangle \langle \psi_{\text{str}}(k)| \otimes |\bar{k}'_R\rangle \langle \bar{k}'_R|, \end{aligned} \quad (28)$$

where by applying Eq. (5) for \mathcal{T}_1 ,

$$|\psi_{\text{str}}(k)\rangle = T_{LR}(k) c_R |L\rangle + c_L |R\rangle. \quad (29)$$

The success probability of this operation is also calculated with $\text{Tr}_{\text{ap}2} \rho_1 = 1$

as

$$P(k_L) = [\eta(k)\langle\psi_{\text{str}}(k)|\psi_{\text{str}}(k)\rangle]_f. \quad (30)$$

It is noticed in Eq. (29) that the initial photonic qubit is transferred to the atomic qubit with slight modification by the factor $T_{LR}(k)$. This quantum-state transfer between the atom and photon combined with projective measurement may be viewed as a sort of one-bit teleportation [26], where the scattering acts as a 2-qubit gate to create the entanglement of atom and photon. The loss term of $|0\rangle\langle 0|$, as seen in Eq. (16), is removed by the photon detection even with $\eta(k) < 1$ (and the negligible dark count). In the ideal case of strong coupling limit with $\lambda_L = \lambda_R$ the initial atomic state $|R\rangle$ in Eq. (25) is swapped entirely to the left-polarized photon $|k_L\rangle$ in the output state via scattering, as discussed in the preceding section. [See Eqs. (3) and (14) with $a_L = 0$ and $a_R = 1$.] Then, the probability for the detection of left-polarized photon $P(k_L)$ in Eq. (30) becomes unity with the full quantum efficiency.

Retrieval:

The retrieval of the photonic qubit is implemented by the scattering of the second photon pulse followed by the detection of the atomic state $|L\rangle$. The resultant output state is given by

$$\begin{aligned} \rho_2 &= \frac{1}{P(L)} \langle L | \mathcal{T}_2 \rho_1 \mathcal{T}_2^\dagger | L \rangle \\ &= \frac{|T_{LR}(k_p)|^2}{P(k_L)P(L)} \int_{-\infty}^{\infty} dk \eta(k) |f(k)|^2 |\phi_{\text{rtr}}(k)\rangle \langle \phi_{\text{rtr}}(k)| \end{aligned} \quad (31)$$

with

$$\begin{aligned} |\phi_{\text{rtr}}(k)\rangle &= \int_{-\infty}^{\infty} dk' f(k') e^{-ik'(t-\tau)} \\ &\quad \times \frac{[T_{LR}(k)c_R|k'_R\rangle + T_{LR}(k')c_L|k'_L\rangle]}{T_{LR}(k_p)}, \end{aligned} \quad (32)$$

where $\text{Tr}_{p2}\rho_2 = 1$ to determine the probability $P(L)$ for the projective measurement of $|L\rangle$. It is found here that for $T_{LR}(k) \approx T_{LR}(k_p)$ in the vicinity of narrow pulse peak $|k - k_p| \lesssim \kappa_p \ll \kappa$, the output state becomes very closed to the desired photon state as retrieval:

$$|\phi_{\text{rtr}}(k)\rangle \approx |\phi_{p2}\rangle = c_L|\bar{k}'_L\rangle + c_R|\bar{k}'_R\rangle. \quad (33)$$

Although some modification is made on the stored atomic qubit, as seen in Eq. (29), it is nearly compensated by the retrieval process, as seen in Eq. (33), hence realizing the almost faithful retrieval of the initial photonic qubit. A trade-off for this high fidelity should be paid rather in the success probability, as will be seen in Eq. (38).

Fidelity and success probability:

The quantum memory operation described so far is summarized (see Fig. 1) as

$$\begin{aligned}
|\phi_{p1}\rangle &= c_L|\bar{k}_L\rangle + c_R|\bar{k}_R\rangle \\
&\mathcal{T}_1 \Downarrow \Pi(k_L) \\
|\psi_{\text{str}}(k)\rangle &= T_{LR}(k)c_R|L\rangle + c_L|R\rangle \\
&\mathcal{T}_2 \Downarrow |L\rangle\langle L| \\
|\phi_{\text{tr}}(k)\rangle &\approx |\phi_{p2}\rangle = c_L|\bar{k}'_L\rangle + c_R|\bar{k}'_R\rangle.
\end{aligned} \tag{34}$$

The fidelity for this sequence of storage and retrieval is evaluated from Eqs. (31) and (32) as

$$\begin{aligned}
F(p1 \rightarrow a \rightarrow p2) &= \langle \phi_{p2} | \rho_2 | \phi_{p2} \rangle \\
&= \frac{[\eta(k)|\langle \phi_{p2} | \phi_{\text{tr}}(k) \rangle|^2]_f}{[\eta(k)\langle \phi_{\text{tr}}(k) | \phi_{\text{tr}}(k) \rangle]_f}.
\end{aligned} \tag{35}$$

Since the quantum efficiency may be taken as constant, $\eta(k) = \eta$, around the pulse peak in a good approximation, it is actually calculated in Eq. (35) as

$$F(p1 \rightarrow a \rightarrow p2) = F_{\text{qm}} + (1 - F_{\text{qm}})|c_L|^4 \geq F_{\text{qm}} \tag{36}$$

depending on $|\phi_{p1}\rangle$. The fidelity of quantum memory is then given by

$$\begin{aligned}
F_{\text{qm}} &= | [T_{LR}(k)]_f |^2 / [|T_{LR}(k)|^2]_f \\
&= \frac{| [\sin(\phi_s(k)/2)]_f |^2}{[|\sin(\phi_s(k)/2)|^2]_f}
\end{aligned} \tag{37}$$

irrespective of the choice of unknown initial state. This fidelity F_{qm} is independent of the ratio λ_L/λ_R . Hence, the present quantum memory works without

relying on the specific relation of the dipole couplings. The net success probability of quantum memory is also calculated from Eq. (31) with $\text{Tr}_{p2}\rho_2 = 1$ as

$$P_{\text{qm}} = P(k_L)P(L) = \eta \left[|T_{LR}(k)|^2 \right]_f = \eta \sin^2 2\xi F_{\text{swap}}, \quad (38)$$

where $\sin 2\xi = 2(\lambda_L/\lambda)(\lambda_R/\lambda)$ depending on λ_L/λ_R . It is noticed here that P_{qm} is intimately related to $F_{\text{swap}} = [\sin^2(\phi_s(k)/2)]_f$ given in Eq. (22).

As explained for $P(k_L)$ in the storage process, the second photon pulse $|\bar{k}'_R\rangle$ in Eq. (25) is swapped ideally to the atomic state $|L\rangle$ with $P(L) = 1$ in the strong coupling limit. Then, the success probability of quantum memory P_{qm} becomes unity together with the fidelity of swapping F_{swap} in Eq. (38) with $\eta = 1$ and $\sin 2\xi = 1$ ($\lambda_L = \lambda_R$). The atomic detection of $|L\rangle$ in Eq. (31) for the retrieval process may be implemented by using the third photon pulse. Specifically, by injecting the photon pulse of $|\bar{k}''_L\rangle$ into the cavity, the $|L\rangle|k''_L\rangle$ component is transformed to $T_{RL}(k'')|R\rangle|k''_R\rangle + T_{LL}(k'')|L\rangle|k''_L\rangle$ via scattering while the $|R\rangle|k''_L\rangle$ one is unchanged. Hence, the polarization detection $\Pi(k''_R)$ after the scattering effectively projects out the $|L\rangle$ component as recovering the initial $|R\rangle$ state with the success probability $\eta [|T_{LR}(k)|^2]_f = P_{\text{qm}}$ where $|T_{RL}(k)| = |T_{LR}(k)|$. Then, if this conditional method is used to detect $|L\rangle$ we make a substitution $P(L) \rightarrow P(L)P_{\text{qm}}$ in Eq. (38), providing P_{qm}^2 (rather than P_{qm}) as the success probability of quantum memory.

In a feasible experiment to perform the present atomic quantum memory, a sufficiently weak coherent light of $|\alpha\rangle$ may be used as an actual single-photon source, though the success probability becomes rather small proportional to $|\alpha|^2$. The photon detection is useful to remove the irrelevant contribution from the vacuum component in $|\alpha\rangle$ as well as that from the atomic loss into the environment. The contributions of more than one photon states in $|\alpha\rangle$ are small enough for $|\alpha|^2 \ll 1$.

4 Efficiency of quantum memory

As seen in Eq. (33), for $T_{LR}(k) \approx T_{LR}(k_p)$ in the vicinity of photon-pulse peak $|k - k_p| \lesssim \kappa_p$ with the sufficiently narrow width $\kappa_p \ll \kappa$, the fidelity of the present quantum memory really approaches unity in Eq. (37):

$$F_{\text{qm}} \approx 1 \quad (\kappa_p \ll \kappa). \quad (39)$$

(In this situation with $\kappa_p \ll \kappa$ the first Markov approximation for the usual input-output relation will be valid essentially, where the atom-photon cou-

plings are assumed to be independent of the frequency k [25].) This high fidelity F_{qm} is rather independent of the details of the spectral profiles $f(k)$ and $\eta(k)$. It is not restricted either by the finite atomic loss, cavity leakage or detunings even in the moderate coupling regime providing $|\sin(\phi_s(k_p)/2)|^2 \sim 0.1$, which is in contrast with F_{swap} in Eq. (23). Therefore, we find that the present method is quite promising for implementing quantum memory with single atoms in cavity QED; the memory operation can be performed almost faithfully in a reasonable range of experimental parameters.

These profitable features of the present atomic quantum memory are really confirmed by a numerical analysis to evaluate the fidelities in comparison with the swapping method. We typically take the Gaussian [G] and Lorentzian [L] profiles, which are given, respectively, by

$$f(k)[\text{G}] = \frac{\exp[-(k - k_p)^2/2\kappa_p^2]}{\sqrt{\sqrt{\pi}\kappa_p}} e^{i(k-k_p)x_0}, \quad (40)$$

$$f(k)[\text{L}] = \frac{\sqrt{\kappa_p/\pi}}{k - k_p + i\kappa_p} e^{i(k-k_p)x_0}. \quad (41)$$

Here, the spectral distribution is specified with the peak position k_p and width κ_p , while the spatial location is given by the factor $e^{i(k-k_p)x_0}$. The coordinate x_0 of the center is supposed to be large enough as $x_0 \gg 1/\kappa_p, l$ so that the initial photon pulse is sufficiently apart from the cavity with length l . Then, the phase shift $\phi_s(k)$ is determined independently of $f(k)$, as given in Eq. (9) [23].

We compare in Fig. 2 the fidelities F_{qm} (upper) of the present quantum memory and F_{swap} (lower) of the swapping depending on $\lambda^2/\kappa\gamma$ typically for $\kappa_p = 0.1\kappa$, $\kappa = 2\gamma$ and (solid): $\delta_e = 0$, $\delta_p = 0$, (dashed): $\delta_e = 5\gamma$, $\delta_p = 0$, (dotted): $\delta_e = 0$, $\delta_p = 0.5\gamma$, respectively. The Gaussian profile is taken here. (Precisely, F_{qm} and F_{swap}^2 should be compared for the storage and retrieval.) We clearly see that this quantum memory works efficiently, achieving the quite high fidelity F_{qm} for reasonable experimental parameters such as $(\gamma, \kappa, \lambda)/2\pi \approx (3\text{MHz}, 6\text{MHz}, 15\text{MHz})$ with $\lambda^2/\kappa\gamma \approx 10$ and $\kappa_p \approx 0.1\kappa \approx 0.6\text{MHz}$ [21,22]. As long as $|\delta_e|, |\delta_p| \lesssim \gamma$ ($\sim 1 - 10\text{MHz}$), the detunings do not provide significant effects on the fidelity F_{qm} for $\kappa_p \lesssim 0.1\kappa$ and $\lambda^2/\kappa\gamma \gtrsim 10$. As for the cavity leakage, similar results are obtained for $\kappa \sim (1 - 10)\gamma$. Its optimal value is $\kappa \sim \gamma$; the smaller κ is the smaller κ_p should be taken for $\kappa_p \lesssim 0.1\kappa$, while the larger κ is the larger λ is required for $\lambda^2/\kappa\gamma \gtrsim 10$. The fidelity of swapping F_{swap} , on the other hand, is obviously lower than F_{qm} . It is rather limited by the finite atomic loss, cavity leakage and detunings, as seen in Eq. (23).

We also show in Fig. 3 the fidelity of quantum memory F_{qm} depending on κ_p/κ

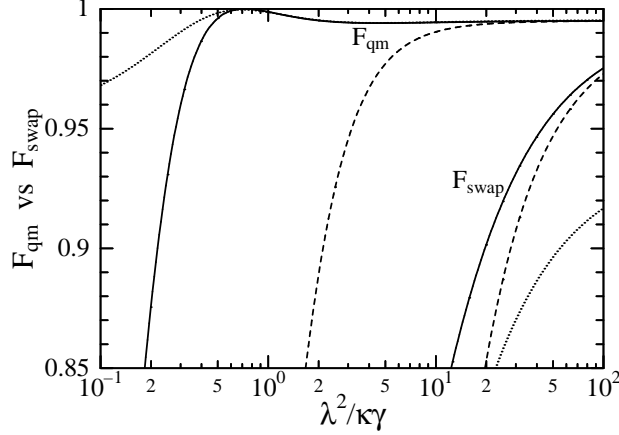


Fig. 2. F_{qm} (upper) and F_{swap} (lower) are compared depending on $\lambda^2/\kappa\gamma$ for $\kappa_p = 0.1\kappa$, $\kappa = 2\gamma$ and (solid): $\delta_e = 0$, $\delta_p = 0$, (dashed): $\delta_e = 5\gamma$, $\delta_p = 0$, (dotted): $\delta_e = 0$, $\delta_p = 0.5\gamma$, respectively. The Gaussian profile is taken here.

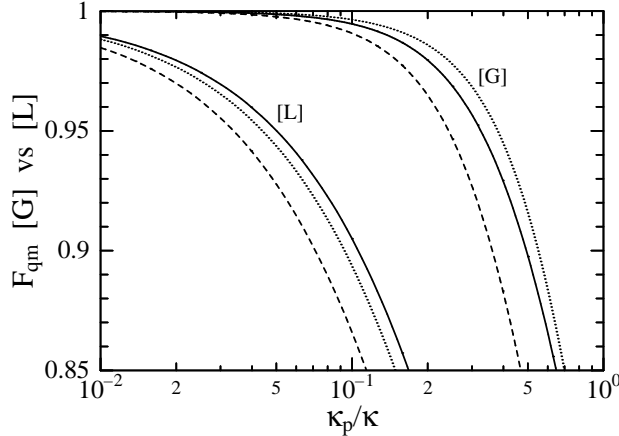


Fig. 3. F_{qm} is shown depending on κ_p/κ for the Gaussian [G] and Lorentzian [L] profiles. Here the parameters are taken as $\lambda^2/\kappa\gamma = 20$, $\kappa = 2\gamma$ and (solid): $\delta_e = 0$, $\delta_p = 0$, (dashed): $\delta_e = 10\gamma$, $\delta_p = 0$, (dotted): $\delta_e = 0$, $\delta_p = 2\gamma$, respectively.

for the Gaussian [G] and Lorentzian [L] profiles. Here the parameters are taken as $\lambda^2/\kappa\gamma = 20$, $\kappa = 2\gamma$ and (solid): $\delta_e = 0$, $\delta_p = 0$, (dashed): $\delta_e = 10\gamma$, $\delta_p = 0$, (dotted): $\delta_e = 0$, $\delta_p = 2\gamma$, respectively. It is observed that the Gaussian profile provides the quite high fidelity as it has the sharp spectral peak compared to the Lorentzian profile. We obtain for instance $F_{\text{qm}} \geq 0.999$ with $\kappa_p/\kappa \lesssim 0.05$ for $\lambda^2/\kappa\gamma = 20$ and $\kappa = 2\gamma$ if the detunings are not too large. The detunings of atom and photon pulse do not provide significant effects, as already seen in Fig. 2. It is also noticed in Fig. 3 that the asymptotic value $F_{\text{qm}} \simeq 0.995$ for $\lambda/\kappa\gamma \gtrsim 10$ in Fig. 2 is really determined by the ratio $\kappa_p/\kappa = 0.1$.

The success probability of quantum memory P_{qm} with $\eta = 1$ is shown in Fig. 4 depending on λ_L/λ_R for $\lambda^2/\kappa\gamma = 1$ (lower), 10 (middle), 100 (upper) with $\kappa_p = 0.1\kappa$, $\kappa = 2\gamma$ and (solid): $\delta_e = 0$, $\delta_p = 0$, (dashed): $\delta_e = 5\gamma$,

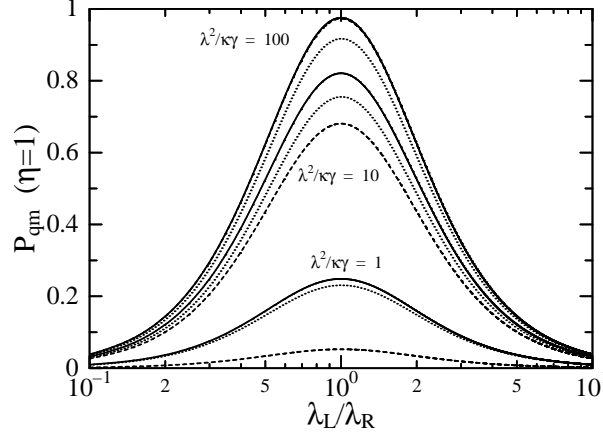


Fig. 4. P_{qm} with $\eta = 1$ is shown depending on λ_L/λ_R for $\lambda^2/\kappa\gamma = 1$ (lower), 10 (middle), 100 (upper) with $\kappa_p = 0.1\kappa$, $\kappa = 2\gamma$ and (solid): $\delta_e = 0$, $\delta_p = 0$, (dashed): $\delta_e = 5\gamma$, $\delta_p = 0$, (dotted): $\delta_e = 0$, $\delta_p = 0.5\gamma$, respectively. The Gaussian profile is taken here. Note that for $\lambda^2/\kappa\gamma = 100$ the solid and dashed lines are almost overlapped.

$\delta_p = 0$, (dotted): $\delta_e = 0$, $\delta_p = 0.5\gamma$, respectively. The Gaussian profile is taken here. (Note that for $\lambda^2/\kappa\gamma = 100$ the solid and dashed lines are almost overlapped.) As seen in Eq. (38), P_{qm} is proportional to the quantum efficiency η of the photon detector, and its dependence on the ratio $\lambda_L/\lambda_R = \tan \xi$ of the dipole couplings is given by the factor $\sin^2 2\xi$, indicating the trade-off for the high F_{qm} . The success probability P_{qm} ($\eta = 1$) is really optimized to coincide with the fidelity of swapping F_{swap} for Λ -type atoms satisfying the condition $\lambda_L = \lambda_R$. The effects of the detunings on P_{qm} appear in the same way as F_{swap} .

We conclude in these calculations that the present method for atomic quantum memory works quite efficiently, achieving the fidelity of almost unity with reasonable experimental parameters.

5 An application: storage of 2-qubit entanglement

As an application of the quantum memory via atom-photon scattering, we here consider storage of 2-qubit entanglement. We prepare two atomic memories and a polarization-entangled pair of photon pulses. Each photon pulse is scattered with the atom inside the respective cavity. Then, for the ideal case of $T_{LR} = T_{RL} = 1$ and $T_{LL} = T_{RR} = 0$, the quantum states of atom pair and photon pair, either entangled or separable, are swapped as

$$\begin{pmatrix} a_{LL} \\ a_{RR} \\ a_{LR} \\ a_{RL} \end{pmatrix}_{a^2} \otimes \begin{pmatrix} c_{LL} \\ c_{RR} \\ c_{LR} \\ c_{RL} \end{pmatrix}_{p^2} \Rightarrow \begin{pmatrix} c_{RR} \\ c_{LL} \\ c_{RL} \\ c_{LR} \end{pmatrix}_{a^2} \otimes \begin{pmatrix} a_{RR} \\ a_{LL} \\ a_{RL} \\ a_{LR} \end{pmatrix}_{p^2} \quad (42)$$

with the basis states $|LL\rangle, |RR\rangle, |LR\rangle, |RL\rangle$ for the atom pair “a²” and $|k_L k_L\rangle, |k_R k_R\rangle, |k_L k_R\rangle, |k_R k_L\rangle$ for the photon pair “p²”. This sort of 2-qubit swapping may be applied to the storage of photonic polarization entanglement as

$$c_{LR}|\bar{k}_L\rangle|\bar{k}'_R\rangle + c_{RL}|\bar{k}_R\rangle|\bar{k}'_L\rangle \Rightarrow c_{RL}|LR\rangle + c_{LR}|RL\rangle. \quad (43)$$

Specifically, by taking the initial state

$$|\Phi_{\text{in}}\rangle = |RR\rangle(c_{LR}|\bar{k}_L\rangle|\bar{k}'_R\rangle + c_{RL}|\bar{k}_R\rangle|\bar{k}'_L\rangle), \quad (44)$$

we obtain the output state via scatterings in the cavities 1 and 2 as

$$|\Phi_{\text{out}}\rangle = \mathcal{T}_1 \mathcal{T}_2 |\Phi_{\text{in}}\rangle. \quad (45)$$

Here the (kk') -component of $|\Phi_{\text{out}}\rangle$ is given by

$$|\Phi_{\text{out}}^{(kk')}\rangle = |\psi_{\text{str}}^{(kk')}\rangle|k_L k'_L\rangle + |RR\rangle|\phi_{LR}^{(kk')}\rangle \quad (46)$$

with

$$|\psi_{\text{str}}^{(kk')}\rangle = T_{LR}(k)c_{RL}|LR\rangle + T_{LR}(k')c_{LR}|RL\rangle, \quad (47)$$

$$|\phi_{LR}^{(kk')}\rangle = T_{RR}(k')c_{LR}|k_L k'_R\rangle + T_{RR}(k)c_{RL}|k_R k'_L\rangle. \quad (48)$$

Then, the fidelity $F(p^2 \rightarrow a^2)$ of the entanglement transfer via swapping is evaluated by tracing over the photon states and the environment denoted by $|0\rangle\langle 0|$. For any choice of the initial state it is bounded as

$$|[T_{LR}]_f|^2 \leq F(p^2 \rightarrow a^2) \leq |[T_{LR}|^2]_f = \sin^2 2\xi F_{\text{swap}}, \quad (49)$$

where $|[T_{LR}]_f|^2 \leq |[T_{LR}|^2]_f$ for $F_{\text{qm}} \leq 1$ in Eq. (37) is considered. For the specific case of $\lambda_L = \lambda_R$ ($\sin 2\xi = 1$), the fidelity is optimally given by F_{swap} of the 1-qubit swapping in Eq. (22), while it is rather below unity for $\lambda_L \neq \lambda_R$.

Alternatively, as done in Sec. 3, we can make actively the photon detection $\Pi(k_L) \otimes \Pi(k'_L)$ on the output state in Eq. (46) to remove the undesired term

$|RR\rangle|\phi_{LR}^{(kk')}\rangle$, so that the trade-off of the success probability is made to obtain the high fidelity. Then, the transfer of the photonic entanglement to the atomic memories can be implemented almost faithfully. The fidelity of this 2-qubit entanglement memory is calculated to be identical to that of the 1-qubit memory in Eq. (37):

$$F_{2\text{qem}} = F_{\text{qm}} \approx 1 \quad (\kappa_p \ll \kappa). \quad (50)$$

Reversely, the entanglement stored in the pair of atomic quantum memories is retrieved by injecting single-photon pulses (which may be separable each other) to the atomic memories. In a feasible experiment for this entanglement transfer, a photon pair from the type-II parametric down-conversion may be used as the input polarization-entangled qubit.

6 Summary

In summary, we have investigated a scheme of atomic quantum memory to store photonic qubits of polarization in cavity QED. The swapping between photonic and atomic qubits can be made via scattering in optical cavities. This swapping operates limitedly in the strong coupling regime for Λ -type atoms with equal dipole couplings. By extending this scheme of atom-photon scattering in cavity QED, we have presented a more feasible and efficient method to implement the quantum memory operation combined with projective measurement. This method works without requiring such a condition on the dipole couplings of Λ -type atoms. The fidelity is significantly higher than that of the swapping, and even in the moderate coupling regime it reaches almost unity by narrowing sufficiently the photon-pulse spectrum. This high performance is rather unaffected by the atomic loss, cavity leakage or detunings, while a trade-off is paid in the success probability for projective measurement.

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